Algorithms and Programming I

Final Exam-Review (Graphs)

Spring 2015

Problem1:

State weather the following statements are true or false.

1. (1) Even though BFS and DFS have the same space complexity, they do not always have the same worst case asymptotic time complexity.

2. If we represent a graph with |V| vertices and Ө (|V |) edges as an adjacency matrix, the worst-case running time of breadth-first search is Ө (|V|2).

3. We can use Dijkstra's algorithm to find the shortest path between two vertices in a graph with arbitrary edge weights.

4. The time it takes to scan all edges for a graph represented as an adjacency matrix is Ө(|V|2)
5. It is often faster to add and remove edges from G when using an adjacency matrix.

6. With adjacency matrices, iterating over all neighbors incident to a vertex v requires only $O(\delta(v))$ time, where $\delta(v)$ is the degree of v.

7. The running time of depth-first search, as a function of |V | and |E|, if the input graph is represented by an adjacency matrix instead of an adjacency list is $O(V^2)$.

8. If the DFS finishing time $f[u] > f[v]$ for two vertices u and v in a directed graph G, and u and v are in the same DFS tree in the DFS forest, then u is an ancestor of v in the depth first tree.

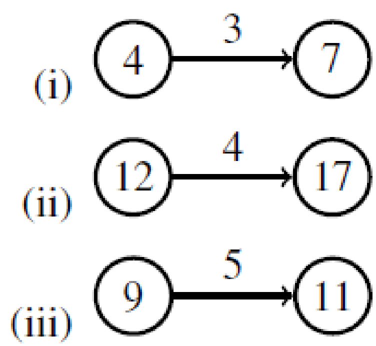
9. The degree of a vertex in an undirected graph is the number of edges incident on it, with a loop being counted twice.

10. A graph with no cycle is acyclic.

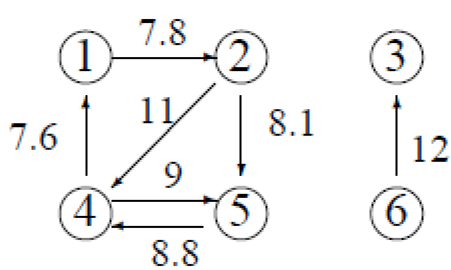
11. DAG is undirected acyclic graph.

Problem 2:

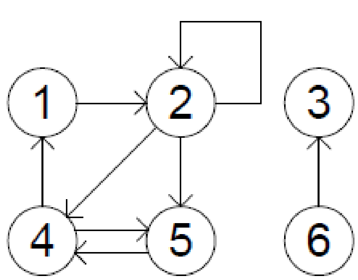
1. What is the result of relaxing the following edges?



2. Draw the graph which represents the following set of vertices and edges: V={1,2,3,4,5,6} , E={{2,5},{1,2},{1,5},{3,6}}
3. Draw the graph which represents the following set of vertices and edges: V={1,2,3,4,5,6} , E={(1,2),(2,4),(4,1),(2,5),(4,5),(5,4),(6,3)}
4. Give the adjacency matrix representation for the following graph:

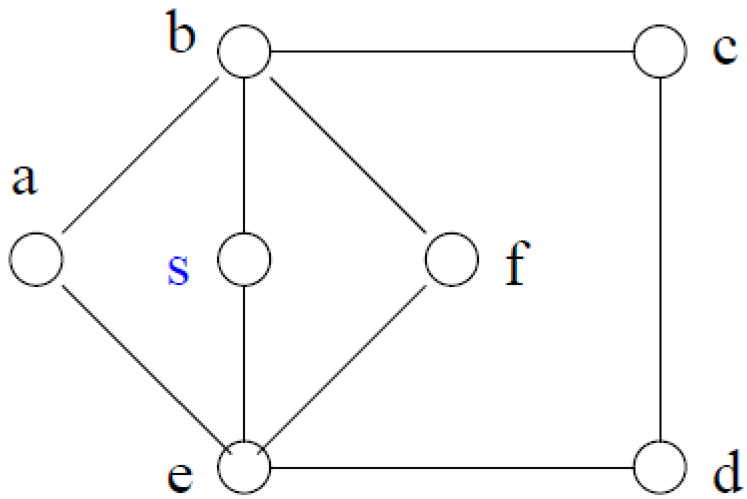


5. Give the adjacency list representation for the following graph:



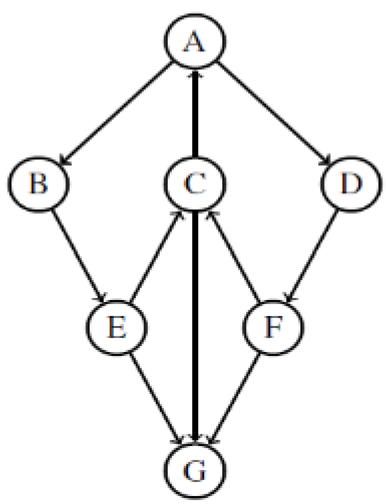
Problem3:

Given the following undirected graph and source vertex, find the distance from s to each vertex u ϵ V and the predecessor pred [u] along a shortest path



Problem4:

Perform a depth-first search on the following graph starting at A. Assume that whenever faced with a decision of which node to pick from a set of nodes, pick the node whose label occurs earliest in the alphabet.



Problem5:

Run Dijkstra's algorithm on the following directed graph, starting at vertex S. What is the order in which vertices get removed from the priority queue? What is the resulting shortest-path tree?